Speed-up of Nonlinear Electromagnetic Field Analysis using Fixed-Point Method

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Abstract — The nonlinear finite element analysis of magnetic fields using the Fixed-Point method requires a lot of number of iterations and long CPU time compared with those using the Newton-Raphson method. On the other hand, the Fixed-Point method has an advantage that a software can be easily programmed by slightly modifying a linear FEM software. Then, we achieved the speed-up of the Fixed-Point method by introducing the derivative of reluctivity (We call this as a modified Fixed-Point method). It is shown that the formulation of the Fixed-Point method using the derivative of reluctivity is almost the same as that of the Newton-Raphson method. The convergence properties of these methods are compared, and it is shown that the convergence of the modified Fixed-Point method is considerably improved compared with the conventional Fixed-Point method.

I. INTRODUCTION

The Fixed-Point method [1,2] has an advantage that the software for nonlinear analysis can be easily obtained by adding a small change to that for linear analysis. In addition, it has an advantage that the convergence can be obtained even for a complicated nonlinear problems such as the analysis considering vector magnetic properties, in which the convergence is sometimes difficult. However, the Fixed-Point method has a problem that it requires a lot of number of iterations and long CPU time compared with those of the Newton-Raphson method. It is reported that the CPU time can be reduced by using a constant reluctivity at several calculations in the beginning of nonlinear iterations [3,4]. However, nearly ten times CPU time is still necessary compared with the Newton-Raphson method.

In this paper, a modified Fixed-Point method, which updates the reluctivity and the derivative of reluctivity at each iteration, is proposed. Furthermore, it is pointed out that the formulation of the Fixed-Point method using the derivative of reluctivity is similar to that of the Newton-Raphson method.

II. METHOD OF ANALYSIS

A. Modified Fixed-Point Method

Fig.1 shows the concept of the nonlinear magnetic field analysis using the modified Fixed-Point method. In this method, the reluctivity ν_{FP} shown in Fig.1 is given as an initial value, and the derivative $\partial \boldsymbol{H}/\partial \boldsymbol{B}$ is updated at each iteration. Namely, at the initial iterations, the linear magnetic field analysis is carried out using the given ν_{FP}, and the flux density $\boldsymbol{B}$' is obtained. Next, the difference $\boldsymbol{H}_{FP}$ between $\boldsymbol{H}(\boldsymbol{B}')$ corresponding to the flux density $\boldsymbol{B}$' on the B-H curve and $\boldsymbol{H}$ on the line of $\partial \boldsymbol{H}/\partial \boldsymbol{B}(\boldsymbol{B}')$ shown in Fig.1 is obtained, and the iteration is carried out until $\boldsymbol{H}_{FP}$ becomes near to zero.

B. Formulation

$\boldsymbol{H}(\boldsymbol{B})$ is given by

$$\boldsymbol{H}(\boldsymbol{B}) = \boldsymbol{\nu}_{FP}\boldsymbol{B} + \boldsymbol{H}_{FP}(\boldsymbol{B}) \tag{1}$$

$\boldsymbol{H}(\boldsymbol{B})$ on the B-H curve is determined by the flux density $\boldsymbol{B}$. The static magnetic field equation can be written as follows in the case of the Fixed-Point method:

$$\nabla \times (\boldsymbol{H} + \boldsymbol{H}_{FP}) = \boldsymbol{J_0} \tag{2}$$

where, $\boldsymbol{J}_0$ is the forced current density.

The difference $\boldsymbol{H}_{FP}(\boldsymbol{B})^{(k+1)}$ at the $(k+1)$-th nonlinear iteration can be obtained by the following equation:

$$\boldsymbol{H}_{FP}{}^{(k+1)} = \boldsymbol{H}(\boldsymbol{B}^{(k)}) - \boldsymbol{\nu}_{FP}\boldsymbol{B}^{(k)} \tag{3}$$

$\boldsymbol{H}(\boldsymbol{B}^{(k)})$ is the magnetic field strength vector on the B-H curve corresponding to the flux density $\boldsymbol{B}^{(k)}$ at the k-th nonlinear iteration. The residual $\boldsymbol{r}_i(\boldsymbol{A})$ is given by

$$r_i(\boldsymbol{A}) = \int \nabla \times \boldsymbol{N}_i \cdot (\nu_{FP} \nabla \times \boldsymbol{A}^{k+1}) dV - \int \boldsymbol{N}_i \boldsymbol{J}_0 dV + \int \nabla \times \boldsymbol{N}_i \cdot \boldsymbol{H}_{FP}{}^k dV \tag{4}$$

where $\boldsymbol{N}_i$ is the interpolation function of i-th edge (edge element is used here). By using Eq.(3), Eq.(4) can be changed as follows:

$$r_i(\boldsymbol{A}) = R_i(\boldsymbol{A}^k) + \int \nabla \times \boldsymbol{N}_i \cdot (\nu_{FP} \nabla \times \delta \boldsymbol{A}) dV \tag{5}$$

where $R_i(\boldsymbol{A}_k)$ is given by

$$R_i(\boldsymbol{A}^k) = \int \nabla \times \boldsymbol{N}_i \cdot (\nu_{FP} \nabla \times \boldsymbol{A}^{k+1}) dV - \int \boldsymbol{N}_i \boldsymbol{J}_0 dV \tag{6}$$

By the way, the residual $r_i(\boldsymbol{A})$ of the nonlinear iteration of the Newton-Raphson method is given by

$$\begin{aligned} r_i(\boldsymbol{A}) &= R_i(\boldsymbol{A}^k) + \delta R \\ &= \int \nabla \times \boldsymbol{N}_i \cdot \boldsymbol{H}^k dV - \int \boldsymbol{N}_i \boldsymbol{J}_0 dV + \int \nabla \times \boldsymbol{N}_i \cdot \delta \boldsymbol{H}^k dV \\ &= R_i(\boldsymbol{A}^k) + \int \nabla \times \boldsymbol{N}_i \cdot \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{B}}(\boldsymbol{B}^k) \cdot \delta \boldsymbol{B}_i{}^k dV \\ &= R_i(\boldsymbol{A}^k) + \int \nabla \times \boldsymbol{N}_i \cdot \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{B}}(\boldsymbol{B}^k) \cdot \nabla \times \delta \boldsymbol{A}^k dV \end{aligned} \tag{7}$$

Eqs. (6) and (7) denote that the formulation of the modified Fixed-Point method is similar to that of the Newton-Raphson method. In the actual calculation, Eq.(4) is used in the calculation of the modified Fixed-Point method. The process of calculation is as follows:

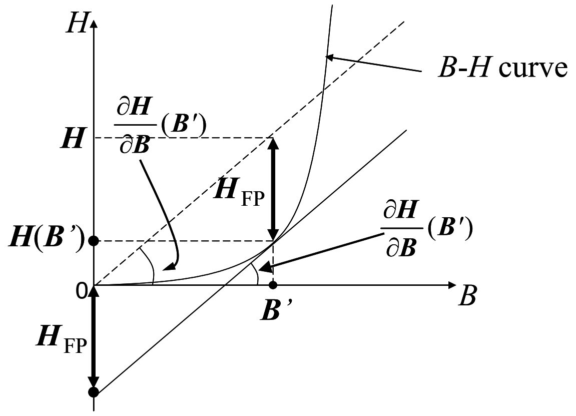


Fig.1 Conceptual diagram of modified Fixed-Point method.

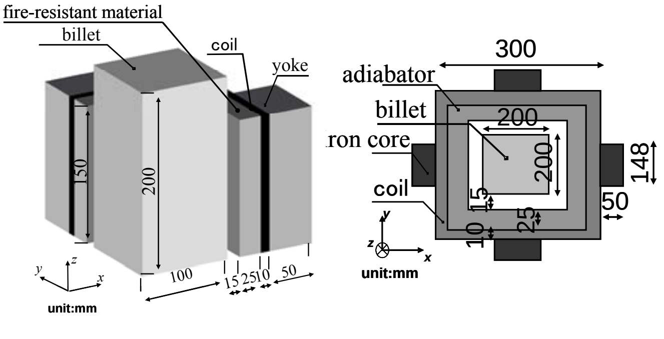
1) The initial value of $\partial \boldsymbol{H}/\partial \boldsymbol{B}(\boldsymbol{B}^0)$ is determined.
2) The difference $\boldsymbol{H}_{\mathrm{FP}}{}^{0}$ is set to zero.
3) $\boldsymbol{B}^k$ is obtained from $\boldsymbol{A}^k$ calculated by Eq.(4).
4) The difference $\boldsymbol{H}_{\mathrm{FP}}{}^{k+1}$ is obtained by Eq.(3) at the first iteration. After the second iteration, $\boldsymbol{H}_{\mathrm{FP}}{}^{(k+1)}$ is calculated by the following equation using the derivative $\partial \boldsymbol{H}/\partial \boldsymbol{B}(\boldsymbol{B}^{(k)})$ and $\boldsymbol{H}(\mathbf{B}^{(k)})$ which is calculated from B-H curve.

$$\boldsymbol{H}_{\mathrm{FP}}{}^{(k+1)} = \boldsymbol{H}(\boldsymbol{B}^{(k)}) - \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{B}}(\boldsymbol{B}^{(k)}) \cdot \boldsymbol{B}^{(k)} \qquad (8)$$

5) The right hand side of Eq.(4) is updated and the process from 3) to 5) is repeated.
6) It is judged to be converged if the change of $\boldsymbol{A}$ is less than the specified small value.

III. ANALYZED MODEL

The modified Fixed-Point method is applied to the analysis of the magnetic field in the billet heater model shown in Fig.2. Analysis domain of the model is 1/8. The material of the yoke is 35H230(non-oriented electrical steel). The numbers of elements and nodes are 107632 and 115101, respectively. The ampere turns of the coil are set as 70000AT (60Hz). The CPU time and number of iteration of the Fixed-Point method(FPM), modified Fixed-Point method(MFPM), Newton-Raphson method using ν-B^2 curve(NRM(B^2)), and Newton-Raphson method using B-H curve(NRM(B)) are compared. For simplicity, the static analysis is carried out in order to compare the performance of each method.



(a) bird's eye biew (b) x-y plane

Fig.2 Model of billet heater.

IV. RESULTS AND DISCUSSION

The comparison of the CPU time and number of iterations is shown in Table I. The convergence property is shown in Fig.3. The convergence criterion is $\boldsymbol{r}_i(\boldsymbol{A}) < 10^{-3}$. The convergence criterion of the ICCG method is chosen as less than 10^{-5}. Intel Core2Duo E8400 @ 3.16GHz, 3GBRAM is used. These results suggests that the performance of MFPM is near to that of NRM.

We can conclude that the modified Fixed-Point method has an advantage that the programming is easy and the effective performance near to the Newton-Raphson method can be obtained.

TABLE I
CPU TIME AND ITERATIONS

Method	CPU Time[sec]	Iterations
NRM(B^2)	559.99	23
NRM(B)	918.55	43
FPM	5912.59	167
MFPM	1408.77	40

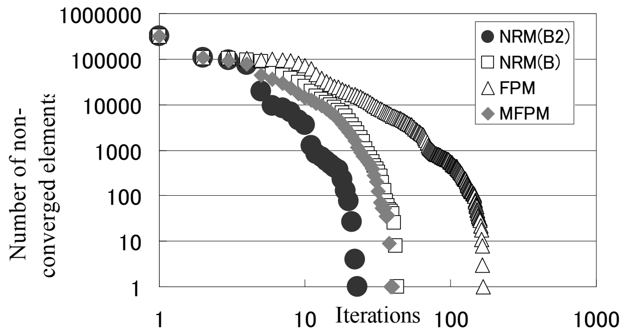


Fig.3 Convergence property.

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